Int. J. Heat Mass Transfer. Vol. 16, pp. 2229–2240. Pergamon Press 1973. Printed in Great Britain

FREE CONVECTIVE CONDENSATION HEAT TRANSFER WITH NONCONDENSABLE GAS ON A VERTICAL SURFACE

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(*Received* 20 *December* 1972)

Abstract—Free convective condensation with noncondensable gas on an isothermal vertical surface is studied under the condition of thermal equilibrium. An analysis is made by use of equations of a liquid film and boundary layers adjoining the liquid film and including small droplets generated by condensation, condensable and noncondensable gases. The calculation is made in the range of 1–99 per cent weight fractions of condensable gas. The result shows that Nusselt number asymptotically approaches those of free convection and film condensation at both extremes, and the phenomenon treated here intermediates free convection and film condensation along a vertical plate.

NOMENCLATURE

a, thermal diffusivity [m^2/s];
C_l, specific heat at constant pressure of liquid [kcal/kg°C];
C_p, specific heat at constant pressure of gas [kcal/kg°C];
D, diffusion coefficient [m^2/s];
E, non-dimensional quantity of latent heat $\equiv L/c_pT$;
g, gravitational acceleration [m/s^2];
Gr, Grashof number $\equiv g\beta\Delta Tx^3/\nu^2$;
h, enthalpy [kcal/kg];
L, latent heat [kcal/kg];
$\dot{m}$, mass flux [kg/m^{2}s];
M, molecular weight;
Nu, Nusselt number;
P, pressure [kg/m^2];
Pr, Prandtl number $\equiv \nu/a$;
$\dot{q}$, heat flux [kcal/m^{2}s];
Sc, Schmidt number $\equiv \nu/D$;
Sh, Sherwood number $\equiv \alpha_M x/D$;
T, temperature [°K];
u, velocity in x direction [m/s];
v, velocity in y direction [m/s];
W, weight concentration of vapour;
x, rectangular co-ordinate [m];
y, rectangular co-ordinate [m];
Y, weight concentration of liquid droplets;
α, heat transfer coefficient [kcal/m^{2}s°C];
α_M, mass transfer coefficient [m/s];
β, thermal coefficient of volumetric expansion [1/°K];
γ, specific weight [kg/m^3];
δ, boundary layer thickness [m];
$\Delta\theta$, temperature difference in liquid film [°C];
λ, thermal conductivity [kcal/ms°C];
μ, viscosity [kg/ms];
ν, kinematic viscosity [m^2/s];
ρ, weight per unit volume [kg/m^3].

Suffixes

g, non-condensable gas;
l, liquid;
n, thermodynamic nonequilibrium condition;
0, gas–liquid interface;
v, condensing gas;
x, function of x;
w, wall;
∞, main flow.

1. INTRODUCTION

NUMEROUS papers have reported on the subject of film condensation heat transfer since the pioneering work of Nusselt in 1916. Since then, it has been known that both heat and mass transfer fluxes decrease greatly when vapour contains a small amount of noncondensable gas.

With the condensation of vapor to a liquid film, noncondensable gas mixed with the vapor close to the liquid film, and constructs a resistive layer which prevents the vapor to move to the film. This is the cause of reduction of heat and mass transfer rates when condensing vapour contains noncondensable gas.

Applying a boundary layer theory to the resistive layer outside the liquid film when vapor includes a great deal of noncondensable gas, Sparrow and others [1] computed an exact solution of free convective condensation heat transfer with noncondensable gas. Also, Rose [2] analyzed the same problem with the boundary layer integration method, and concluded that the approximate equation agreed fairly well to the exact solution. In these analysis, mass conservation equations for noncondensable gas and condensing vapour are used and no consideration is given to the relation between the concentration and the temperature of the condensing gas, the so called "saturation condition". The assumptions they adopted are nearly correct when the relative humidity of the vapor in the main flow is low (high superheat), but are irrelevant when the influences of noncondensable gas are dominant, i.e. the condensation of low superheat or saturated vapor. In other words, in the analysis reported so far, there is a thermodynamical nonequilibrium, that is, the concentration of the condensing vapor at a certain point in a boundary layer is larger than the saturated concentration of condensing vapor corresponding to the temperature of that point (super saturation). From a thermodynamical standpoint, such a state is unstable, but the following state is stable and practical, in which the super saturated part of the vapors is dispersed in the vapor as liquid droplets, and the concentration of vapor is kept at the saturation concentration of the temperature at that point.

From this point of view, in this paper we analyse the condensing phenomena of a saturated vapor with a noncondensable gas along a vertical surface, assuming there is a liquid film and a gas phase boundary layer containing small droplets in which the thermodynamic equilibrium is kept between the vapor and the liquid droplets, and we study the effects of the noncondensable gas on condensation heat transfer.

2. THEORETICAL ANALYSIS

2.1 *Background of the theory*

First, we consider the two-phase boundary layer covering the liquid film which is the basis of this study. Figure 1 shows the relation between temperature and concentration of condensing vapor and of liquid droplets in the boundary layer where the temperature varies. Curve 1 shows a thermodynamic equilibrium relation between the temperature and vapor concentration for a steam–air mixture where steam whose weight fraction is 0·972 at 100°C is cooled under the constant total pressure. Due to the saturation condition, with the decrease of temperature, the concentration of steam decreases and a part of steam condensates (liquid droplets), whose weight fraction Y is shown by curve 2 in Fig. 1. In the past other analysis of condensation heat transfer with a noncondensable gas, conservation equations for energy and condensing gas

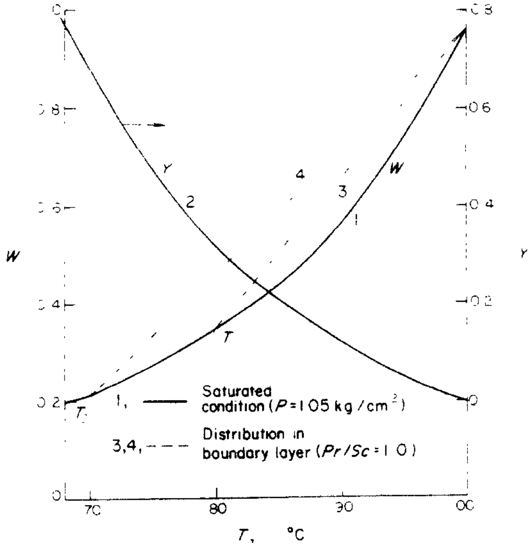
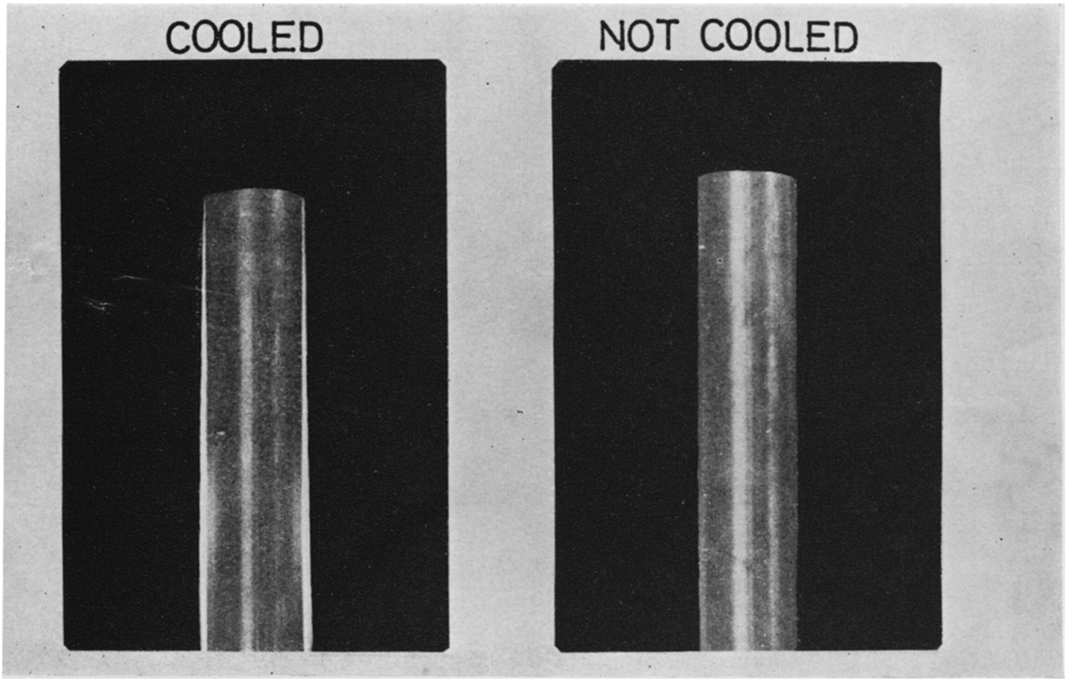


FIG. 1. Relation between weight fraction of condensable gas and temperature.



Fɪɢ. 2. Photographs of cooled and not-cooled cylinders

are independently analyzed. So the relation between temperature T and concentration W in the layer is defined by the straight lines 3 or 4 in Fig. 1, when the temperature at gas–liquid film interface shows T(80°C), T(70°C). In this case we assume Sc/Pr (Sc: Schmidt number, Pr: Prandtl number) is 1, which means the same boundary layer thicknesses of energy and concentration. Therefore, in the past other analysis, there existed thermodynamical nonequilibrium corresponding to the difference between curve 1 and curves 3, 4. Furthermore, straight lines 3 and 4 change into curves and approach curve 1 when Sc/Pr is smaller than 1, and become even lower than curve 1 for much smaller Sc/Pr. In this case, the usual analysis seems to be more stable in terms of thermodynamics. But the critical value of the ratio Sc/Pr which makes curves 3, 4 lower than curve 1 depends on the kind of vapor and the temperature difference between the main steam and the gas–liquid film interface. When Sc/Pr is larger than unity, curves 3 and 4 are deeply curved in the middle and the degree of nonequilibrium becomes greater. As shown in Fig. 1, the greater the temperature difference between the main stream and the gas–liquid interface, the greater the degree of thermodynamical nonequilibrium.

In the case of free convective condensation on a vertical surface, which condition will really occur, a thermodynamical equilibrium condition or nonequilibrium one? If the thermodynamical equilibrium condition is realized in the boundary layer, a part of the supersaturated vapor is condensed and many liquid droplets appear, so if we light them, the scattered light should be visible. Two cylinders having an outer diameter 30ϕ, and a thickness of 1 mm are exposed to a light, and the photos are shown in Fig. 2. One of the cylinders is cooled inside by ice and located in a room temperature, and the other is not cooled. For the first one, the photo is shown in the left picture of Fig. 2, and we can see the scattered light by the minute liquid droplets which are condensed in the gas around the cooled cylinder. Therefore, even in the case of the condensation of steam contained in normal air, it is more realistic to consider that the thermodynamical equilibrium condition is satisfied in the boundary layer. In this study, we analyze the free convective condensation heat transfer around a vertical flat surface assuming the thermal equilibrium relation as shown by curve 1, in Fig. 1, between the temperature T and the concentration of condensing vapor W in a noncondensable gas.

2.2 *Fundamental equation*

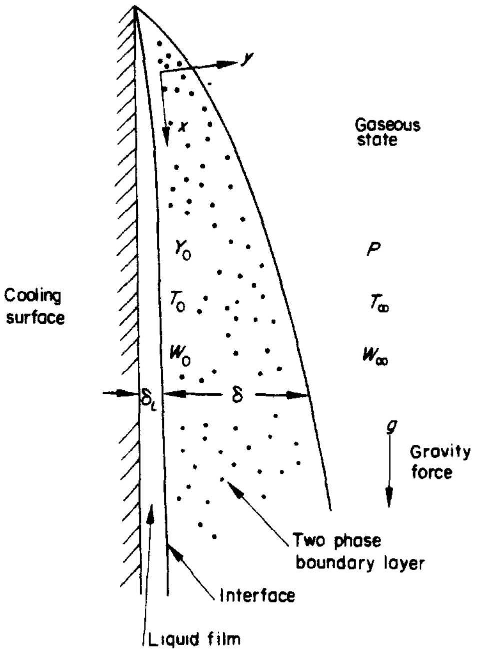


FIG. 3. Schematic model of physical situation.

An analytical model used in the analysis is schematically shown in Fig. 3. We assume a thin liquid film, having a thickness of δ_l on a vertical cooled plate, and a gas–liquid droplet coexisting two phase boundary layer of thickness δ outside the liquid film where the saturation condition is satisfied. A coordinate x is along the interface between the liquid film and the boundary layer and a coordinate y is perpendicular to x. Subscripts 0 and ∞ mean values at $y = 0$ and

$y = \infty$, and the subscripts g, v and l express values of noncondensable gas, condensing vapor and liquid, respectively. For the analysis, the following assumptions are adopted:

(1) Shearing stress by gas flow has no effect on the motion of the liquid film.
(2) Variation of the liquid film thickness along x direction does not effect the gas–liquid droplet coexisting (two phase) boundary layer.
(3) Boundary layer approximation can be applied to the two phase boundary layer.
(4) Specific weight of gas is negligible as compared with that of liquid.
(5) Volume concentration of liquid droplets is negligible.
(6) Total pressure is constant, and the density change by the temperature variation is only considered in the buoyancy term of a momentum equation.
(7) Physical properties are constant and values at the wall temperature are used, and those of gas mixture are determined by algebraic means of weight fraction.
(8) Saturation condition is satisfied over the whole area in the two-phase boundary layer.

The propriety of the above mentioned assumptions from (1) through (7) have already been examined in many papers [3], so it will be omitted here.

Concerning the liquid film, the Nusselt's solution can be used as the assumptions (1) and (2). A mass flux $\dot{m}$ and a heat flux $\dot{q}$ across the interface between the liquid film and the boundary layer and a velocity at the interface u_0 are expressed as functions of the liquid film thickness δ_l as follows;

$$\dot{m} = \frac{g\gamma_l}{\nu_l}\delta_l^2\frac{\mathrm{d}\delta_l}{\mathrm{d}x} \tag{1}$$

$$\dot{q} = \frac{\lambda_l\Delta\theta}{\delta_l} - \frac{3}{8}\frac{\gamma_l c_l\Delta\theta}{\nu_l}\delta_l^2\frac{\mathrm{d}\delta_l}{\mathrm{d}x} \tag{2}$$

$$u_0 = \frac{g}{2\nu_l}\delta_l^2 \tag{3}$$

where $\Delta\theta$ represents the temperature drop in the liquid film.

In the two phase boundary layer, the average velocity of the gas phase and the velocity of the liquid droplets are considered to be equal, because liquid droplets of enormously small diameter are made by condensation from the gas phase, and the velocity of the gas phase is slow. In this case, the fundamental equations for the gas–liquid droplets coexisting boundary layer are shown as follows:

Continuity equation of gas and liquid droplet mixture is

$$\frac{\partial}{\partial x}\{(\rho + \rho_l)u\} + \frac{\partial}{\partial y}\{(\rho + \rho_l)v\} = 0 \tag{4}$$

where ρ means the weight per unit volume, and the letters without subscript express the values for the mixture of non-condensable gas and condensing vapor. Mean velocities of mixture and mean specific weight are,

$$\rho = \rho_g + \rho_v$$

$$\rho u = \rho_g u_g + \rho_v u_v, \qquad \rho v = \rho_g v_g + \rho_v v_v. \tag{5}$$

The velocities of the components of non-condensable gas are given by Fick's law using a binary diffusion coefficient D

$$\rho_g(u - u_g) = \rho D\frac{\partial}{\partial x}\left(\frac{\rho_g}{\rho}\right),$$

$$\rho_g(v - v_g) = \rho D\frac{\partial}{\partial y}\left(\frac{\rho_g}{\rho}\right). \tag{6}$$

But the diffusion velocity in the x direction can be neglected by assumption (3), so we get $u_g = u_v = u$.

The continuity equation of noncondensable gas is,

$$\frac{\partial}{\partial x}(\rho_g u) + \frac{\partial}{\partial y}(\rho_g v_g) = 0. \tag{7}$$

The momentum equation of gas and liquid droplets mixture is,

$$\frac{\partial}{\partial x}\{(\rho + \rho_l)u^2\} + \frac{\partial}{\partial y}\{(\rho + \rho_l)uv\} = (\rho + \rho_l - \rho_\infty)g + \frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y}. \tag{8}$$

The energy equation of the mixture is

$$\frac{\partial}{\partial x}\{(\rho_g h_g + \rho_v h_v + \rho_l h_l)u\} + \frac{\partial}{\partial y}\{\rho_g h_g v_g + \rho_v h_v v_v + \rho_l h_l v\} = \frac{\partial}{\partial y}\lambda\frac{\partial}{\partial y}T \quad (9)$$

where h means enthalpy, and are given as follows: L is a latent heat for evaporation.

$$h_g = C_{pg}(T - T_\infty), h_v = C_{pv}(T - T_\infty), h_l = C_l \times (T - T_\infty) - L. \quad (10)$$

From assumption (8), a partial pressure of condensing vapor P_v is given as a function for temperature only.

$$p_v = f(T). \quad (11)$$

We introduce new variables, the weight fractions of condensing vapor W and of the liquid droplets Y,

$$W = \frac{\rho_v}{\rho} = \frac{\rho_v}{\rho_g + \rho_v}, \qquad Y = \frac{\rho_l}{\rho} = \frac{\rho_l}{\rho_g + \rho_v}. (12)$$

Using assumption (6), the weight fraction W can be related to the partial pressure of the condensing vapor P_v,

$$W = \frac{(P_v/P)(M_v/M_g)}{1 - (P_v/P)(1 - M_v/M_g)} \quad (13)$$

where M is a molecular weight. Upon considering these, equations (6)–(9), and (11) are rewritten as follows:

$$\frac{\partial}{\partial x}(1 + Y)u + \frac{\partial}{\partial y}(1 + Y)v = 0 \quad (14)$$

$$\frac{\partial}{\partial x}(1 - W)u + \frac{\partial}{\partial y}(1 - W)v = -D\frac{\partial^2 W}{\partial y^2} \quad (15)$$

$$\frac{\partial}{\partial x}(1 + Y)u^2 + \frac{\partial}{\partial y}(1 + Y)uv = \left\{Y + \beta(T_\infty - T) + \frac{(M_g - M_v)(W_\infty - W)}{(1 - W_\infty)M_v + W_\infty M_g}\right\}g + \nu\frac{\partial^2 u}{\partial y^2} \quad (16)$$

$$\frac{\partial}{\partial x}[\{C_{pg} + W(C_{pv} - C_{pg}) + C_l Y\}(T_\infty - T) + LY]u + \frac{\partial}{\partial y}[\{C_{pg} + W(C_{pv} - C_{pg}) + C_l Y\}(T_\infty - T) + LY]v = -\frac{\lambda}{\rho}\frac{\partial^2 T}{\partial y^2} + D(C_{pv} - C_{pg})(T_\infty - T)\frac{\partial W}{\partial y} \quad (17)$$

$$W = F(T). \quad (18)$$

The boundary condition at $y = 0$.

$$u = u_0, v_g = 0\left(v = -\frac{D}{1 - W_0}\frac{\partial W}{\partial y_{y=0}}\right)$$
$$T = T_0, W = W_0 = F(T_0) \quad (19)$$

and thosd at $y = \infty$,

$$u = 0, \; T = T_\infty, \; W = W_\infty = F(T_\infty), \; Y = 0. \quad (20)$$

As the relation between W and T in equation (18) is generally nonlinear, the analytical solution of these equations is barely obtainable. Therefore, in this study, the boundary layer integral method is used. The following conditions are used to determine the distribution of variables.

$$y = 0; \; Y = Y_0$$
$$y = \delta_x; \; v = v_\delta, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial W}{\partial y} = 0, \quad \frac{\partial Y}{\partial y} = 0. \quad (21)$$

So the distribution form are assumed as follows,

$$u = u_x\left(\varphi + \frac{y}{\delta_x}\right)\left(1 - \frac{y}{\delta_x}\right)^2, \quad Y = Y_0\left(1 - \frac{y}{\delta_x}\right)^2$$
$$\frac{T_\infty - T}{\Delta T} = \left(1 - \frac{y}{\delta_x}\right)^n; \quad \frac{W_\infty - W}{\Delta W} = \left(1 - \frac{y}{\delta_x}\right)^2$$
$$u_x\varphi = u_0, \quad \Delta T = T_\infty - T_0, \quad \Delta W = W_\infty - W_0 \quad (22)$$

where the subscript x means the function of x, and δ_x is the thickness of the boundary layer.

The saturation condition by equation (18) cannot be satisfied all through the boundary layer by the distribution in equation (22). As the quantities near $y = 0$ are most important in the integral method, an index of the temperature distribution n in equation (22) is determined by a differential of T and Y with y at $y = 0$ as to satisfy the saturation condition in equation (18). That is, in Fig. 1, when T_2 is the interface temperature of the liquid film, the distribution assumed in equation (22) is coincident with curve 1, at both sides and in the gradient at T_2. As curve 1 is a very simple shape, this approximation is very good in all areas. By the method above mentioned,

$$n = 2\frac{(\Delta W/\Delta T)}{(\partial W/\partial T)_{y=0}} = 2\left[\left(\frac{1}{W^2}\right)\left(\frac{P_v}{P}\right)^2\left(\frac{M_v}{M_g}\right)\right.$$
$$\left.\times\left(\frac{\Delta W}{\Delta T}\right)\Big/\left(\frac{1}{P}\frac{\partial P_v}{\partial T}\right)_{y=0}\right] \geqq 2. \tag{23}$$

The index n is larger than 2, and the 2nd derivative of T can exist at $y = \delta_x$ as a finite value. The distributions in equation (22) are substituted into equations (14)–(17), and these equations are integrated from $y = 0$ to $y = \delta_x$. The dependency of u_x and δ_x on x are determined so that the power of x coincides in each term of equations (14)–(17). In this calculation, we get the results that φ is constant, $u_x = ax^{\frac{1}{2}}$ and $\delta_x = bx^{\frac{1}{4}}$. v_δ is eliminated from the integrated equations and they are arranged as follows.

$$R\{V_\infty - Y_0\} = V_0(1 + Y_0) \tag{24}$$

$$R\left[\frac{30\{\varphi(n+4)+1\}}{(6\varphi+1)(n+3)(n+4)}\frac{C_{p\infty}}{C_{p0}}\right.$$
$$+\frac{30\{\varphi(n+6)+1\}}{(6\varphi+1)(n+5)(n+6)}$$
$$\left.\times\left\{\left(\frac{C_{pg}-C_{pv}}{C_{p0}}\right)\Delta W + \frac{C_l}{C_{p0}}Y_0\right\} + EY_0\right]$$
$$= V_0\left[\frac{C_{pv}}{C_{p0}} + \frac{C_l}{C_{p0}}Y_0 + EY_0\right] + \frac{n}{Pr_0} \tag{25}$$

$$R^2\left[\frac{100(1+7\varphi+21\varphi^2)}{21(6\varphi+1)^2}\times\right.$$
$$\left.+\,Y_0\,\frac{125(1+9\varphi+36\varphi^2)}{63(6\varphi+1)^2}\right]$$
$$+R\left[\frac{10(1-2\varphi+V_0Y_0\varphi)}{(6\varphi+1)}\right] = \left(\frac{Gr}{4}\right)\left(\frac{\delta_x}{x}\right)^4$$
$$\left[\frac{1}{n+1} + \frac{\Delta W}{3\beta\Delta T}\left(\frac{M_g - M_v}{M}\right) + \frac{Y_0}{3\beta\Delta T}\right] \tag{26}$$

where

$$R = \left(\frac{6\varphi+1}{40}\right)ab^2,\quad C_{p0} = (1 - W_0)C_{pg}$$
$$+ W_0C_{pv},\quad C_{p\infty} = (1 - W_\infty)C_{pg} + W_\infty C_{pv}$$

$$V_\infty = \frac{\Delta W}{1 - W_0},\quad V_0 = \frac{\Delta W}{1 - W_0}\frac{2}{Sc},$$

$$E = \frac{L}{C_{p0}\Delta T},\quad M = (1 - W_\infty)M_v + W_\infty M_g$$

and the heat flux and mass flux across the gas and liquid film interface, and the velocity of the interface are derived as follows,

$$\dot{m} = (1 + Y_0)\nu_0 V_0\left(\frac{1}{\delta_x}\right) \tag{27}$$

$$\dot{q} = \rho_0 C_{p0}\nu_0\Delta T\left(\frac{n}{Pr_0} + V_0E\right)\left(\frac{1}{\delta_x}\right) \tag{28}$$

$$u_0 = \frac{40\varphi}{(6\varphi+1)}R\frac{\nu_0 x}{\delta_x^2}. \tag{29}$$

Equating equations (27)–(29) to equations (1)–(3), we get three equations, and the six unknown variables are determined from these three equations and equations (24) and (26). In the calculation, φ is given, and eliminating R from equations (24) and (25), we get a quadratic equation of Y_0. The positive root of this equation is the needed one and the two negative solutions correspond to the super-heated state in a boundary layer, namely curves 3 or 4 are under the curve 1 in Fig. 1, and this case is not considered in this analysis. By using the positive solution of Y_0, R and δ_x are obtained from equations (24) and (26), and δ_l, $\Delta\theta$, φ are also obtained. Using the value of φ thus obtained, the same procedure is carried until φ is converged.

The heat flux and the mass flux are calculated from equations (27) and (28), and Nusselt number and Sherwood number are determined as follows.

$$\frac{Nu}{(Gr/4)^{\frac{1}{4}}} = (n + Pr_0EV_0)A^{\frac{1}{4}} \tag{30}$$

$$\frac{Sh}{(Gr/4)^{\frac{1}{4}}} = \frac{2(1 + Y_0)}{(1 - W_0)}A^{\frac{1}{4}} \tag{31}$$

where

$$A = \frac{\left[\dfrac{1}{n+1} + \dfrac{\Delta W}{3\beta\Delta T}\left(\dfrac{M_g - M_v}{M}\right) + \dfrac{Y_0}{3\beta\Delta T}\right]}{R^2\left[\dfrac{100(1 + 7\varphi + 21\varphi^2)}{21(6\varphi + 1)^2} + Y_0\dfrac{125(1 + 9\varphi + 36\varphi^2)}{63(6\varphi + 1)^2}\right] + R\dfrac{10(1 - 2\varphi + V_0Y_0\varphi)}{(6\varphi + 1)}}. \tag{32}$$

3. RESULT

3.1 *In the case of a small temperature difference*

In order to study the essential differences between this investigation where the thermodynamic equilibrium condition is satisfied, and the various analytical studies in the past where the equilibrium condition is not considered, we obtain the analytical results of both conditions for a small temperature difference. In the case of a small temperature difference, we can neglect the interface velocity and condensing velocity at the interface ($u_0 = 0 \quad v_0 = 0$). To study the physical meaning of the analytical solutions, we assume $C_l = C_{pg} = C_{pv}$ and $\Delta W = Y_0 = V_0 = V_\infty = 0(\varepsilon)$. The fundamental equations are shown without considering the thermodynamical equilibrium condition, in the case $\varepsilon_n \ll 1$ where $1 - \varepsilon_n$ is the ratio of a thermal boundary layer thickness δ_T to a concentration boundary layer thickness δ_w, that is δ_T/δ_w.

Continuity equation:

$$R_n = \frac{2}{Sc}(1 + 2\varepsilon_n) \tag{33}$$

Energy equation:

$$R_n = \frac{2}{Pr} \tag{34}$$

Momentum equation:

$$R_n^2\left(\frac{100}{21}\right) + R_n(10) = \left\{\frac{1}{3} + \frac{\Delta W}{3\beta\Delta T} \times \left(\frac{M_g - M_v}{M}\right)(1 - \varepsilon_n)\right\}\left(\frac{Gr}{4}\right)\left(\frac{\delta_{xn}}{x}\right)^4 \tag{35}$$

where the subscript n means the nonequilibrium condition. From equations (33) and (34),

$$\varepsilon_n = \frac{1}{2}\left(1 - \frac{Pr}{Sc}\right) \quad \text{or} \quad \frac{Pr}{Sc} = 1 - 2\varepsilon_n. \tag{36}$$

In the case of the thermal equilibrium condition, n in equation (23) is rewritten as follows by the Tayler expansion.

$$n = 2\left(\frac{(\Delta W/\Delta T)}{(\partial W/\partial T)}\right)_{y=0} = 2\left[1 + \frac{1}{2}\left(\frac{(\partial^2 W/\partial T^2)}{(\partial W/\partial T)}\right)_{y=0}\Delta T\right]. \tag{37}$$

Therefore, in the case of a small temperature difference, it is regarded as $n = 2$ in the first order approximation.

The fundamental equations for the thermal equilibrium condition are derived from equations (24)-to (26) for the small temperature difference.

$$R\left(1 - \frac{Y_0}{\Delta W}\right) = \frac{2}{Sc} \tag{38}$$

$$R\left(1 - \frac{L}{C_pT}\frac{\Delta W}{\beta\Delta T}\frac{Y_0}{\Delta W}\right) = \frac{2}{Pr} \tag{39}$$

$$R^2\left(\frac{100}{21}\right) + R(10) = \left[\frac{1}{3} + \frac{\Delta W}{3\beta\Delta T}\frac{M_g - M_v}{M} + \frac{\Delta W}{3\beta\Delta T}\frac{Y_0}{\Delta W}\right]\left(\frac{Gr}{4}\right)\left(\frac{\delta_x}{x}\right)^4. \tag{40}$$

The ratios of the mass flux and the heat flux in equilibrium to those in a nonequilibrium condition are equal to the ratio of the Nusselt number and the Sherwood numbers in equilibrium to those in a nonequilibrium condition respectively, and they are shown as follows,

$$\frac{\dot{m}}{\dot{m}_n} = \frac{Sh}{Sh_n} = (1-\varepsilon_n)\left(\frac{\delta_{xn}}{\delta_x}\right) \tag{41}$$

$$\frac{\dot{q}}{\dot{q}_n} = \frac{Nu}{Nu_n} = \left\{\frac{1 + (L/C_pT)\Gamma(Pr/Sc)}{1 + (L/C_pT)\Gamma(Pr/Sc)(1+\varepsilon_n)}\right\} \times \left(\frac{\delta_{xn}}{\delta_x}\right) \tag{42}$$

where

$$\Gamma = \Delta W/\beta\Delta T = T\Delta W/\Delta T.$$

The boundary layer thickness ratio of nonequilibrium to the equilibrium condition case is given from equations (33)–(40)

$$\frac{\delta_{xn}}{\delta_x} = \left[\left\{\frac{1 + \Gamma\left(\dfrac{M_g - M_v}{M}\right) + \Gamma\dfrac{(1 - Pr/Sc)}{1 + (L/C_pT)\Gamma(Pr/Sc)}}{1 + \Gamma(M_g - M_v)/M \times (1 - \varepsilon_n)}\right\} \times \left\{\frac{1 + \dfrac{21}{20}Pr}{\left(1 + \dfrac{(1 - Pr/Sc)(L/C_pT)\Gamma}{1 + (L/C_pT)\Gamma(Pr/Sc)}\right)^{-2} + \dfrac{21}{20}Pr\left(1 + \dfrac{(1 - Pr/Sc)(L/C_pT)\Gamma}{1 + (L/C_pT)\Gamma(Pr/Sc)}\right)^{-1}}\right\}\right]^{\frac{1}{4}}. \tag{43}$$

We get the values of equations (41) and (42) by use of equation (43). In equations (41)–(43), the parameter Γ plays a very important role. Considering $n = 2$, and assuming the saturation equation as $P_v = P_{v0}\exp(-L/R_vT)$, we get the relation of

$$\Gamma = \frac{L}{R_vT}\frac{M}{M_v}W, \tag{44}$$

where R_v is a gas constant of condensing vapor.

For the case of $\Gamma \ll 1$ and $\Gamma \gg 1$, equations (41) and (42) are arranged as follows,

$$\Gamma \ll 1 \left(\frac{\delta_{xn}}{\delta_x}\right) = 1. \quad \frac{\dot{m}}{\dot{m}_n} = \frac{Sh}{Sh_n} = \frac{1}{2}\left(1 + \frac{Pr}{Sc}\right).$$

$$\frac{\dot{q}}{\dot{q}_n} = \frac{Nu}{Nu_n} = 1 \tag{45}$$

$$\Gamma \gg 1 \left(\frac{\delta_{xn}}{\delta_x}\right) = \left[\frac{2}{(1 + Pr/Sc)}\left(\frac{Sc}{Pr}\right)^2 \left(\frac{1 + 1{\cdot}05Pr}{1 + 1{\cdot}05Sc}\right)\right]^{\frac{1}{4}}, \quad \frac{\dot{m}}{\dot{m}_n} = \frac{Sh}{Sh_n} = \frac{\dot{q}}{\dot{q}_n} = \frac{Nu}{Nu_n}$$

$$= \left[\frac{1}{8}\left(1 + \frac{Pr}{Sc}\right)^3 \left(\frac{Sc}{Pr}\right)^2 \left(\frac{1 + 1{\cdot}05Pr}{1 + 1{\cdot}05Sc}\right)\right]^{\frac{1}{4}}. \tag{46}$$

$\Gamma \ll 1$ means that the temperature difference in the boundary layer plays more important roles than the concentration difference; therefore, the driving force of free convective motion is given by the temperature difference and the thermal boundary layer thickness of the nonequilibrium case is equal to the boundary layer thickness of the equilibrium one. For these reasons, the heat fluxes of each case are equal, but the mass fluxes are different. The mass flux of the equilibrium case is smaller than that of the nonequilibrium case by

$$\left(\frac{\dot{m}_n - \dot{m}}{m_n}\right) = \frac{1}{2}\left(1 - \frac{Pr}{Sc}\right),$$

because the thickness of the concentration boundary layer in equilibrium is δ_x, while that for nonequilibrium is $(1-\varepsilon_n)\delta_{xn} = (1/2 + Pr\ 2Sc)$

δ_{xn} from equation (36). But it is noted that these relations can only be adapted to the case of $Sc > Pr$, because in the case of $Sc < Pr$, the condensing vapor is always in a superheated state in the boundary layer. In an actual calculation, the sign of the weight fraction of the liquid droplets Y_0 at the interface shows the realization of the gas–liquid droplets coexisting boundary layer.

In the case of $\Gamma \gg 1$, the concentration difference plays an important role, and the driving force of the convective motion is caused by the concentration difference, and the heat flux to the liquid film is produced by a liberation of the latent heat due to the mass flux. Therefore, the heat flux ratio $\dot{q}/\dot{q}_n$ is equal to the mass flux ratio $\dot{m}/\dot{m}_n$, and is a function of Pr and Sc.

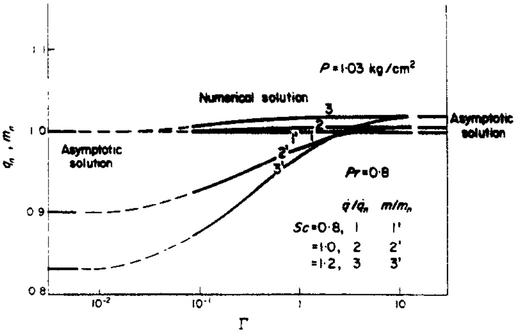


FIG. 4. Relations between $\dot{m}/\dot{m}_n$. $\dot{q}/\dot{q}_n$ and Γ for $\Delta T \to 0$.

The variations of $\dot{m}/\dot{m}_n$ and $\dot{q}/\dot{q}_n$ against Γ are shown in Fig. 4 for $\Delta T \to 0$, as Sc is changed as a parameter and Pr is 0·8. In Fig. 4, the solid line represents the exact solutions of equations (30)–(32), and the extreme limits at both sides are obtained from (45) and (46). From Fig. 4, we can understand that the accuracy of the approximate solutions is very good. The used physical properties, except Pr and Sc, are the values of air and steam.

The dotted lines below $\Gamma = 0{\cdot}08$ are corresponding to the wall temperature below 0°C, and liquid film cannot exist.

3.2 *In the case of a wide temperature difference range*

Considering steam as the condensing vapor, and air as the noncondensable gas, we make a numerical calculation by use of the physical properties of steam and air. These results are given as follows: In Fig. 5, the values of $Nu/(Gr/4)^{\frac{1}{4}}$ to the weight fraction W_∞ of the steam in the main flow, are indicated, where Nu represents a local Nusselt number $(\alpha x/\lambda)$ and Gr is a Grashof number $(g\beta\Delta T x^3/\nu^2)$. The solid lines in Fig. 5 indicate the value when the interface temperature T_0 is changed as a parameter. The increase of Nu with the increase of T_0 is caused by the increase of the mass flux due to the increase of the weight fraction of steam W_0 at the interface. The reverse of this tendency at large W_∞ (at the point of $W_\infty = 0{\cdot}65$ and

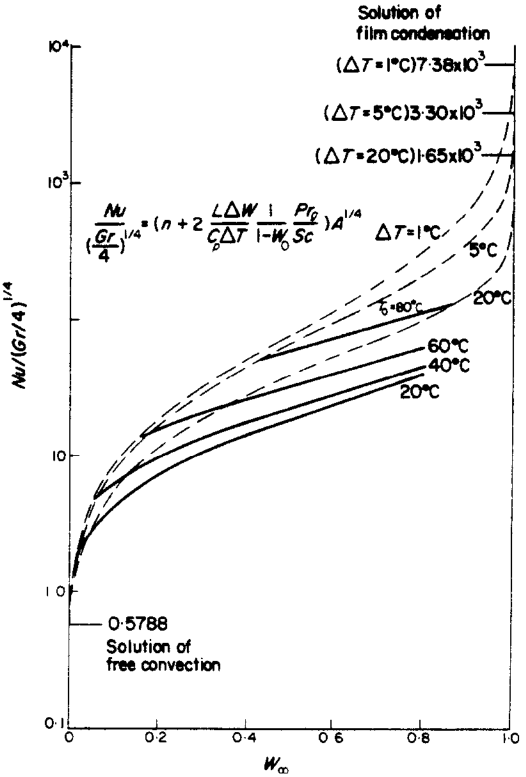


FIG. 5. Relation between $Nu/(Gr/4)^{\frac{1}{4}}$ and weight fraction W_∞ of the steam in main flow.

$T_0 = 20°\text{C}, 40°\text{C}$) is caused by the remarkable effects of the thermal equilibrium condition. The dot-dash lines in Fig. 5 show the changes of Nu by W_∞, and ΔT is varied as a parameter. $Nu/(Gr/4)^{\frac{1}{4}} = 0{\cdot}5788$ at $W_\infty = 0$ corresponds to the exact solution of the free convective heat transfer without condensation, and the values of $7{\cdot}36 \times 10^3$, $3{\cdot}30 \times 10^3$ and $1{\cdot}65 \times 10^3$ at $W_\infty = 1$ are corresponding to the solutions of a film-wise condensation by Nusselt when the temperature differences are 1, 5 and 20°C respectively. In our analysis, we calculate from $W_\infty = 0{\cdot}01$ to $W_\infty = 0{\cdot}99$, and these results tend to approach the well known solutions mentioned above at both extreme limits as shown by dotted lines. It reserves special emphasis that the addition of the solution of free convection and that of liquid film condensation to our result makes Fig. 5 more meaningful as it not only makes the relation between the two extreme cases clear but includes many interesting cases in the field of heat transfer. One interesting case shown in this figure is the increase of the heat flux by the presence of small condensing vapor at a region near $W_\infty = 1$, while the other one is its decrease by the presence of little condensable gas at a region near $W_\infty = 1$. The values of $Nu/(Gr/4)^{\frac{1}{4}}$ and $Sh/(Gr/4)^{\frac{1}{4}}$ are shown relating to the temperature difference between the interface and the main flow, in Fig. 6 when T_∞ is a parameter. In the past other analysis using a nonequilibrium condition, their values decrease with the increase of ΔT, but in this study using a saturation condition, this tendency to decrease is only in a range of small ΔT and they increase at high ΔT. The weight fraction of the liquid droplets on the interface Y_0 shows the effect of the saturation conditions clearly. A relation of Y_0 and ΔT is shown in Fig. 7, when T_∞ is varied as a parameter. In the large temperature difference region, Y_0 does not depend on T_∞ and increases exponentially with ΔT. In the air–steam system, Pr is larger than Sc ($Pr = 0{\cdot}7$, $Sc = 0{\cdot}6$), so in the small temperature difference region Y_0 is negative.

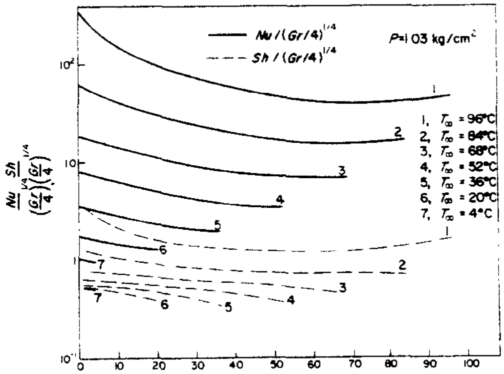


FIG. 6. Variation of $Nu/(Gr/4)^{\frac{1}{4}}$ and $Sh/(Gr/4)^{\frac{1}{4}}$ with temperature difference between interface and main flow.

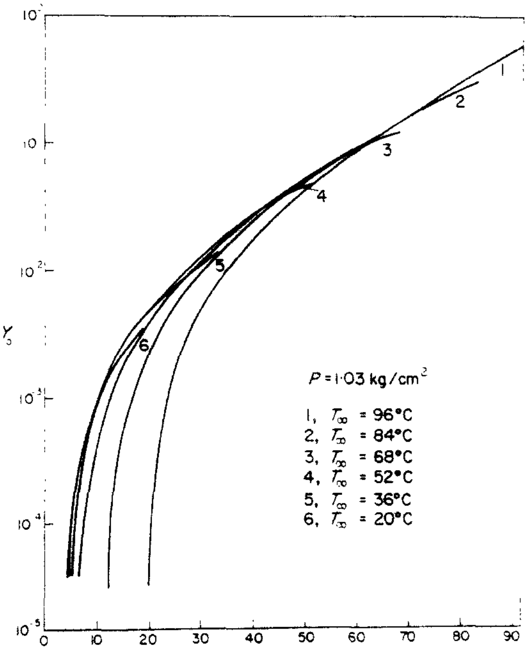


FIG. 7. Relation between Y_0 and ΔT.

In order to indicate the variations of the heat and the mass flux caused by the saturation condition, the relations of $\dot{m}/\dot{m}_n$ and $\dot{q}/\dot{q}_n$ to ΔT are shown in Figs. 8 and 9. The tendency of $\dot{q}/\dot{q}_n$ to increase with the increase of ΔT is nearly independent of T_∞, and $\dot{q}/\dot{q}_n$ becomes

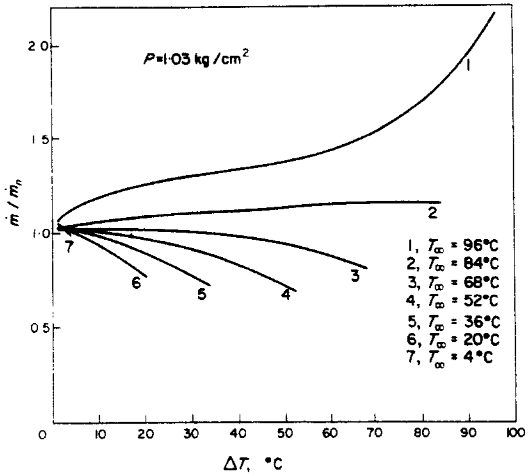


FIG. 8. Relation between $\dot{m}/\dot{m}_n$ and ΔT.

nearly twice when ΔT is large. On the other hand, $\dot{m}/\dot{m}_n$ decreases with the increase of ΔT in a small T_∞ region, and it increases with the increase of ΔT in a large T region. In this region $\dot{m}/\dot{m}_n$ is nearly equal to $\dot{q}/\dot{q}_n$ because in the large T_∞, i.e. the large W_∞ region, heat is mainly transfered by the latent heat due to the mass flux as mentioned above.

Usually, the heat flux near $W_\infty = 1$ is expressed by the ratio to that of a filmwise condensation by Nusselt. The heat flux of the filmwise condensation on a vertical plate was given by Rohsenow [4] who improved the theory of Nusselt and the result is as follows,

$$\dot{q}_{Nu} = \left\{\frac{\rho_l g\{L + (\frac{3}{8})C_p\Delta T\}(\lambda_l\Delta T)^3}{4\nu_l x}\right\}^{\frac{1}{4}}. \qquad (47)$$

In Fig. 10, the relations between the ratio of $\dot{q}$ obtained in this analysis to $\dot{q}_{Nu}$ in equation (47) and ΔT are shown. In a small ΔT region, $\dot{q}/\dot{q}_{Nu}$ is reduced with the decrease of ΔT, because $\dot{q}$ is proportional to $(\Delta T)^{\frac{5}{4}}$, and $\dot{q}_{Nu}$ is proportional to $(\Delta T)^{\frac{3}{4}}$. On the other hand, in a large ΔT region, the decrease of $\dot{q}/\dot{q}_{Nu}$ with the increase of ΔT is caused by a reduction of the heat flux $\dot{q}$ in this analysis, as shown in Fig. 6.

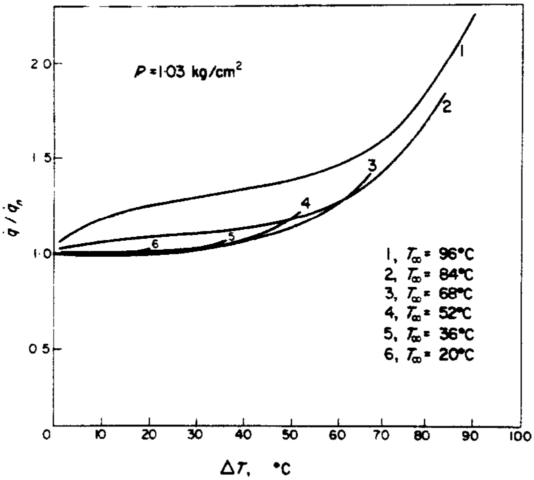


FIG. 9. Relation between $\dot{q}/\dot{q}_n$ and ΔT.

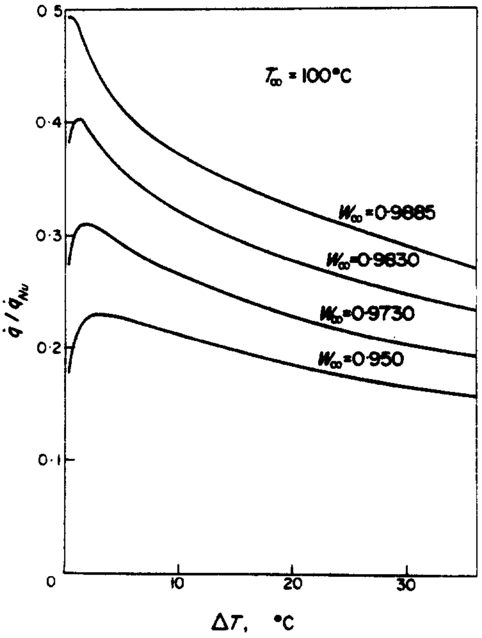


FIG. 10. Relation between $\dot{q}/\dot{q}_{Nu}$ and ΔT.

In [1], Sparrow said 10–20 per cent difference existed between the theory and the experiment, but in his analysis, there is no consideration of the saturation condition. The difference will be explained by taking into account the thermodynamical equilibrium condition proposed in this study.

4. CONCLUSIONS

From the study on the free convective condensation heat transfer around a vertical flat plate containing non-condensable gas, the following results are obtained:

(1) It should be pointed out that the thermodynamical equilibrium condition has not been considered in the temperature boundary layer in the usual analysis. This study indicates that the equilibrium condition has to be considered in the actual phenomena.

(2) According to the above consideration, the analysis is carried out by use of a boundary layer integral method under the condition of a thermal equilibrium condition for steam and air mixture. The same calculation is made with no consideration of the equilibrium condition, and the results are discussed.

(3) The effect of the concentration of the condensable vapor on the heat flux are discussed. and the solutions are obtained, which gradually approach the solution for the free convection heat transfer at $W \rightarrow 0$ as the one extreme limit, and they approach as the other extreme limit the solutions for the film condensation heat transfer at $W \rightarrow 1$. Therefore, the heat transfer performance in the whole region of the free condensable heat transfer is shown in one figure having and relating free convection and film condensation as two extreme limits.

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TRANSFERT THERMIQUE POUR UNE CONDENSATION SUR SURFACE VERTICALE EN PRESENCE DE GAZ INCONDENSABLE

Résumé—On étudie la condensation par convection libre sur une surface verticale isotherme dans les conditions d'équilibre thermique et en présence de gaz incondensable. L'analyse est faite à partir des équations du film liquide et de la couche limite adjacente, laquelle contient des petites gouttes générées par condensation. Le calcul est conduit pour des fractions massiques de gaz condensable comprises entre 1 et 99 pour cent. Les résultats montrent que la valeur du nombre de Nusselt approche asymptotiquement aux deux extrêmes celle de la convection naturelle et celle de la condensation en film, et que le phénomène étudié ici est intermédiaire.

WÄRMEÜBERGANG BEI DER KONDENSATION AN EINER SENKRECHTEN FLÄCHE UNTER FREIER KONVEKTION UND IN GEGENWART VON INERTGASEN

Zusammenfassung—Bei thermischem Gleichgewicht wurde die Kondensation unter freier Konvektion und in Gegenwart von Inertgas an einer isothermen senkrechten Fläche untersucht. Die Analyse schliesst ein Gleichungen des Flussigkeitsfilms und der Grenzschichten an diesem Film und berücksichtigt die Erzeugung kleiner Tropfen sowie kondensierbares wie nichtkondensierbares Gas. Die Berechnungen erstrecken sich auf den Bereich von 1–99 Gewichtsprozent kondensierbaren Gases. Das Ergebnis zeigt, dass sich die Nusselt-Zahl asymptotisch den Werten für freie Konvektion und Filmkondensation in beiden Extremfällen nähert. Das hier behandelte Phänomen liegt zwischen den Fällen freier Konvektion und Filmkondensation an einer senkrechten Wand.

СВОБОДНАЯ КОНВЕКЦИЯ ПРИ КОНДЕНСАЦИИ НА ВЕРТИКАЛЬНОЙ СТЕНКЕ ПРИ НАЛИУИИ НЕКОНДЕНСИРУЮЩЕГОСЯ ГАЗА

Аннотация—При условии теплового равновесия исследовалась свободноконвективная конденсация на изотермической вертикальной поверхности при наличии неконденсируюзегося газа. Анализ проводился с помощью уравнений жидкой пленки и примыкающего к ней пограничного слоя, и учитывал мелкие капли, образующиеся при конденсации, а также конденсирующиеся и неконденсирующиеся газы. Расчеты проводились в диапазоне весовых концентраций конденсирующихся газов от 1 до 99%. Результаты показывают, что число Нуссельта асимптотически приближается к его значениям при свободной конвекции и пленочной конденсации в обоих пределах и что рассматриваемые здесь явления находятся в промежутке между свободной конвекцией и пленочной конденсацией вдоль вертикалной пластины.